The OC curve of p chart

The operating-characteristic (or OC) function of the fraction nonconforming control chart is a graphical display of the probability of incorrectly accepting the hypothesis of statistical control (i.e., a type II or β-error) against the process fraction nonconforming. The OC curve provides a measure of the **sensitivit**y of the control chart—that is, its ability to detect a shift in the process fraction nonconforming from the nominal value $\bar{p}$ to some other value p. The probability of type II error for the fraction nonconforming control chart may be computed from

$$\begin{aligned}\beta &= P\{\hat{p} < \text{UCL}|p\} - P\{\hat{p} \leq \text{LCL}|p\} \\ &= P\{D < n\text{UCL}|p\} - P\{D \leq n\text{LCL}|p\}\end{aligned}$$

Since D is a binomial random variable with parameters n and p, the β-error defined in equation above can be obtained from the cumulative binomial distribution. Note that when the LCL is negative, the second term on the right-hand side of equation) should be dropped.

Table below illustrates the calculations required to generate the OC curve for a control chart for fraction nonconforming with parameters $n = 50$, LCL $= 0.0303$, and UCL $= 0.3697$. Using these parameters, equation becomes

$$\beta = P\{D < (50)(0.3697)|p\} - P\{D \le (50)(0.0303)|p\}$$
$$= P\{D < 18.49|p\} - P\{D \le 1.52|p\}$$

However, since D must be an integer, we find that

$$\beta = P\{D \le 18|p\} - P\{D \le 1|p\}$$

The OC curve is plotted in Fig. below

Calculations[a] for Constructing the OC Curve for a Control Chart for Fraction Nonconforming with $n = 50$, LCL = 0.0303, and UCL = 0.3697

p	$P\{D \le 18 \mid p\}$	$P\{D \le 1 \mid p\}$	$\beta = P\{D \le 18 \mid p\} - P\{D \le 1 \mid p\}$
0.01	1.0000	0.9106	0.0894
0.03	1.0000	0.5553	0.4447
0.05	1.0000	0.2794	0.7206
0.10	1.0000	0.0338	0.9662
0.15	0.9999	0.0029	0.9970
0.20	0.9975	0.0002	0.9973
0.25	0.9713	0.0000	0.9713
0.30	0.8594	0.0000	0.8594
0.35	0.6216	0.0000	0.6216
0.40	0.3356	0.0000	0.3356
0.45	0.1273	0.0000	0.1273
0.50	0.0325	0.0000	0.0325
0.55	0.0053	0.0000	0.0053

[a]The probabilities in this table were found by evaluating the cumulative binomial distribution. For small p ($p < 0.1$, say) the Poisson approximation could be used, and for larger values of p the normal approximation could be used.

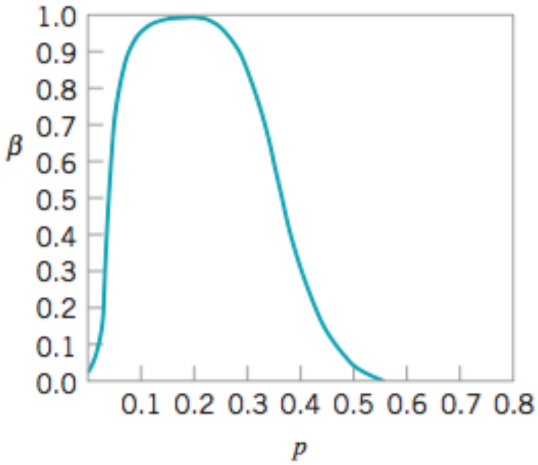


FIGURE Operating-characteristic curve for the fraction nonconforming control chart with $\bar{p} = 0.20$, LCL = 0.0303, and UCL = 0.3697.

We may also calculate average run lengths (ARLs) for the fraction nonconforming control chart.

To illustrate, consider the control chart for fraction nonconforming used in the OC curve calculations in Table 7.6. This chart has parameters $n = 50$, UCL = 0.3697, LCL = 0.0303, and the center line is $p = 0.20$. From Table 7.6 (or the OC curve in Fig. 7.11) we find that if the process is in control with $p = \bar{p}$, the probability of a point plotting in control is 0.9973. Thus, in this case $\alpha = 1 - \beta = 0.0027$, and the value of ARL_0 is

$$ARL_0 = \frac{1}{\alpha} = \frac{1}{0.0027} \simeq 370$$

Therefore, if the process is really in control, we will experience a false out-of-control signal about every 370 samples. (This will be approximately true, in general, for any Shewhart control chart with three-sigma limits.)

Now suppose that the process shifts out of control to $p = 0.3$. Table indicates that if $p = 0.3$, then $\beta = 0.8594$. Therefore, the value of ARL_1 is

$$ARL_1 = \frac{1}{1-\beta} = \frac{1}{1-0.8594} \simeq 7$$

and it will take about seven samples, on the average, to detect this shift with a point outside of the control limits.

The Operating-Characteristic Function of c and u chart

The operating-characteristic (OC) curves for both the *c* chart and the *u* chart can be obtained from the Poisson distribution.

For the *c* chart, the OC curve plots the probability of type II error β against the true mean number of defects c. The expression for β is

β= P$\{x < UCl \ \vdots/ \ \ c\}$ $-P\{\{x < LCl \ \vdots/ \ \ c\}\}$ i.e the probability that the shift is not deducted in the first subsequent sample when there is shift in the value of c ,where x is a Poisson random variable with parameter c. Note that if the LCL < 0 the second term on the right-hand side of equation should be dropped. We will generate the OC curve for the ***c* chart** .

Consider the control chart for c with LCL = 6.48 and the UCL = 33.22.Here $\bar{c}$ is 19.85 . So the expression for β is

$$\beta = P\{x < 33.22|c\} - P\{x \le 6.48|c\}$$

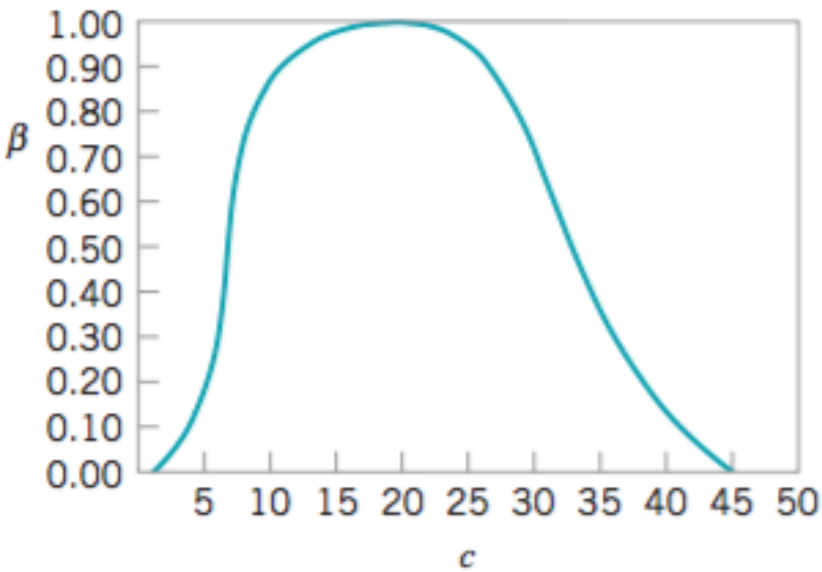
Since the number of nonconformities must be integer, this is equivalent to

$$\beta = P\{x \le 33|c\} - P\{x \le 6|c\}$$

These probabilities are evaluated in Table below and the OC curve is also shown below.

Calculation of the OC Curve for a c Chart with UCL = 33.22 and LCL = 6.48

c	$P\{x \le 33\|c\}$	$P\{x \le 6\|c\}$	$\beta = P\{x \le 33\|c\} - P\{x \le 6\|c\}$
1	1.000	0.999	0.001
3	1.000	0.966	0.034
5	1.000	0.762	0.238
7	1.000	0.450	0.550
10	1.000	0.130	0.870
15	0.999	0.008	0.991
20	0.997	0.000	0.997
25	0.950	0.000	0.950
30	0.744	0.000	0.744
33	0.546	0.000	0.546
35	0.410	0.000	0.410
40	0.151	0.000	0.151
45	0.038	0.000	0.038



OC curve of a *c* chart with LCL = 6.48 and UCL = 33.22.

The OC curve of U chart

For the *u* chart, we may generate the OC curve from

$$\begin{aligned}\beta &= P\{x < \text{UCL}|u\} - P\{x \le \text{LCL}|u\} \\ &= P\{c < n\text{UCL}|u\} - P\{c \le n\text{LCL}|u\} \\ &= P\{n\text{LCL} < x \le n\text{UCL}|u\} \\ &= \sum_{x=\langle n\text{LCL}\rangle}^{[n\text{UCL}]} \frac{e^{-nu}(nu)^x}{x!}\end{aligned}$$

where *n*LCL denotes the smallest integer greater than or equal to *n*LCL and

[nUCL] denotes the largest integer less than or equal to nUCL. The limits on the summation in equation follow from the fact that the total number of nonconformities observed in a sample of n inspection units must be an integer. Note that n need not be an integer.